

Dynamical Interpretation of the Nucleon-Nucleon Interaction and Exchange Currents in the Large N_C Limit

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Abstract

Expression of the nucleon-nucleon interaction to order $1/N_C$ in terms of Fermi Invariants allows a dynamical interpretation of the interaction and leads to a consistent construction of the associated interaction currents to order $1/N_C$. The numerically significant components of 4 different modern realistic phenomenological interaction models are shown to admit very similar meson exchange interpretations in the large N_C limit. Moreover the ratio of the volume integrals of the leading, next-to-leading and next-to-next leading order terms in these interaction models is roughly 300:5-10:0.1, which corresponds fairly well to the ratios of $1/N_C^2$ between the terms that would be suggested by the $1/N_C$ expansion if $N_C = 3$. The N_C dependence of the electromagnetic and axial interaction currents that are associated with these interaction components is derived and compared to that of the corresponding single nucleon currents.

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1 Introduction

The most promising *QCD*-based perspective on nuclear structure may very well be that provided by the large color number (N_C) limit, which allows a systematic series expansion in $1/N_C$ for hadronic observables, where the first few terms appear to capture the key phenomenological features of the structure of the baryons [1]. The leading terms of the $1/N_C$ series expansion of the components of the nucleon-nucleon have been shown to correspond well with the strongly coupled boson exchange terms in a phenomenological boson exchange model for the interaction [2]. While the large N_C limit itself cannot determine the radial behavior of the potential components, this result nevertheless provides a systematic, if approximate connection between nuclear phenomenology and *QCD*.

Here it will be shown that in fact all the most commonly employed modern realistic phenomenological interaction models a) allow an interpretation in terms of phenomenological boson exchange and b) that the numerically significant (or well determined) components of these interactions correspond very well to the leading terms in the $1/N_C$ expansion of the nucleon-nucleon interaction components. The next-to-leading order terms are shown to be numerically very small in comparison to the terms of leading order. The ratio of the volume integrals of the leading, next-to-leading and next-to-next leading order terms in these interaction models is roughly 300:5-10:0.1, which corresponds fairly well to the ratios of $1/N_C^2$ between the terms that would be suggested by the $1/N_C$ expansion if $N_C = 3$.

Finally the corresponding $1/N_C$ scaling factors of the associated two-nucleon exchange (or interaction) currents are derived. The axial exchange charge operator that is associated with long range pion exchange is shown to scale as N_C^0 , as the single nucleon axial charge, whereas those that are associated with the Fermi invariant components of the interaction scale as $1/N_C$ or higher powers in $1/N_C$. The electromagnetic exchange currents scale with two additional powers of $1/N_C$ in comparison with the single nucleon current. The axial exchange current scales as $1/N_C^3$, and is thus smaller by $1/N_C^4$ than the axial current of the nucleon. These scaling factors correspond well with established nuclear phenomenology.

The key to the dynamical interpretation of a phenomenological interaction model is to express it in terms of the 5 Fermi invariants S, V, T, A, P . These invariants form a unique set, that is independent of particle momenta, and allow an interpretation of the interaction in terms of linear combinations of scalar, vector, axial vector and pseudoscalar exchange mechanisms, without the need for explicit specification of the radial behavior of the potential functions. Moreover all of these invariants define unique and consistent interaction current operators [3, 4], for which $1/N_C$ expansions follow from those of the corresponding potentials, as will be shown below.

The utility of the large N_C limit of *QCD* in nuclear phenomenology has emerged only gradually, the first indication being the a priori surprising phenomenological quality of the description of selected observables of nucleons, hyperons and nuclei in terms of Skyrme's topological soliton

model [5] and its generalizations. These models represent one realization of the large N_C limit, in which the baryons appear as topologically stable solutions of nonlinear chiral meson field theories [6]. In particular it was found that Skyrme's product ansatz for the baryon number 2 system led to a phenomenologically satisfactory description of the longest range components of the isospin dependent part of the nucleon-nucleon interaction [7, 8]. This result is in fact very natural as these components of the interaction are of leading order in the $1/N_C$ expansion for which the Skyrme model should give generically adequate results.

The modern phenomenological nucleon-nucleon interaction models that are considered here are the $V18$ [9], the $CD-Bonn$ [10], the $Nijmegen(93)$ [11] and the $Paris$ [12] potentials. All of these contain a long range pion exchange tail, and have mostly or purely phenomenological short and intermediate range terms. Their components in the S, V, T, A, P representation are nevertheless remarkably similar in strength, and if parametrized in terms of single meson exchange give rise to "effective" meson-nucleon coupling strengths, which also are very similar.

In the large N_C limit the transformation between the phenomenological potential components given in the spin representation and the representation in terms of Fermi invariants reveals a new result in that leading order parts of the isospin dependent scalar and vector potentials have to be equal in magnitude with opposite sign. If interpreted in terms of meson exchange this suggests that a_0 and ρ meson exchange have equal strength in the large N_C limit.

In section 2 the expressions for the Fermi invariant decomposition for the large N_C limit of the nucleon-nucleon interaction are derived. In section 3 the corresponding components for the 4 interaction models considered are calculated. All of these are then shown to admit a meson exchange interaction with very similar coupling strengths. Their well determined components are then shown to be consistent with the interaction form suggested by the leading terms in the $1/N_C$ expansion. In sections 4 and 5 the N_C scaling factors of the corresponding electromagnetic and axial interaction current operators are derived. A concluding discussion completes the paper.

2 The Nucleon-Nucleon Interaction in terms of Fermi Invariants in the large N_C Limit

2.1 Phenomenological interactions

The nucleon-nucleon interaction is commonly expressed in terms of the following set of Galilean invariant spin- and isospin operators:

$$V_{NN} = \sum_i^5 [\tilde{v}_j^+ + \tilde{v}_j^- \vec{\tau}^1 \cdot \vec{\tau}^2] \tilde{\Omega}_j, \quad (1)$$

where the coefficients \tilde{v}_j^\pm are scalar functions and the spin operators $\tilde{\Omega}_j$ are defined as

$$\begin{aligned}\tilde{\Omega}_C &= 1, \quad \tilde{\Omega}_{LS} = \vec{L} \cdot \vec{S}, \quad \tilde{\Omega}_T = S_{12}, \\ \tilde{\Omega}_{SS} &= \vec{\sigma}^1 \cdot \vec{\sigma}^2, \quad \tilde{\Omega}_{LS2} = \frac{1}{2} \{ \vec{\sigma}^1 \cdot \vec{L}, \vec{\sigma}^2 \cdot \vec{L} \}_+.\end{aligned}\quad (2)$$

Because the little group of transformations, which leave the square of the 4-velocity of the two-particle system invariant is the same for Galilean and Poincaré transformations, the interaction (1) is invariant under Poincaré transformations.

The power of the leading term in the $1/N_C$ expansion for these potential components \tilde{v}_j^\pm , and therefore their order in $1/N_C$, has been shown to be the following [2]:

$$\begin{aligned}\tilde{v}_C^+ \tilde{\Omega}_C, \quad \tilde{v}_T^- \tilde{\Omega}_T, \quad \tilde{v}_{SS}^- \tilde{\Omega}_{SS} &\sim \mathcal{O}(N_C), \\ \tilde{v}_C^- \tilde{\Omega}_C, \quad \tilde{v}_2^\pm \tilde{\Omega}_{LS}, \quad \tilde{v}_T^+ \tilde{\Omega}_T, \quad \tilde{v}_{SS}^+ \tilde{\Omega}_{SS}, \quad \tilde{v}_{LS2}^- \tilde{\Omega}_{LS2} &\sim \mathcal{O}(1/N_C), \\ \tilde{v}_{LS2}^+ \tilde{\Omega}_{LS2} &\sim \mathcal{O}(1/N_C^3).\end{aligned}\quad (3)$$

A simple derivation of these scaling relations is given below. The numerically significant, or “large”, potential components should therefore, on the basis of their order in $1/N_C$, be the isospin independent central, and isospin dependent spin-spin and tensor components of the potential. This corresponds completely to established nuclear structure phenomenology, as the first of these terms corresponds to the short range repulsive and intermediate range attractive force components, and the latter two correspond to components of the long range pion exchange interaction. These are also the force components that are readily derived in the Skyrme model [7].

While the 5 linearly independent potential invariants $\tilde{\Omega}_j$ are sufficient to describe the two-nucleon amplitude for nucleons on their mass shell, they are not dynamically transparent. For a dynamical interpretation, which allows a connection to field theory, reexpression of the interaction in terms of the 5 Fermi invariants S, V, T, A, P is required. These are defined as [13]

$$\begin{aligned}S &= 1, \quad V = \gamma_\mu^1 \gamma_\mu^2, \quad T = \frac{1}{2} \sigma_{\mu\nu}^1 \sigma_{\mu\nu}^2, \\ A &= i \gamma_5^1 \gamma_\mu^1 i \gamma_5^2 \gamma_\mu^2, \quad P = \gamma_5^1 \gamma_5^2.\end{aligned}\quad (4)$$

The nucleon-nucleon interaction, when expressed in terms of the Fermi invariants, takes the form

$$V_{NN} = \bar{u}(p'_1) \bar{u}(p'_2) [v_j^+ + v_j^- \vec{\tau}^1 \cdot \vec{\tau}^2] F_j u(p_1) u(p_2), \quad (5)$$

where the F_j , $j = 1 \dots 5$ represent the Fermi invariants in the order S, V, T, A, P .

To find the relation between the potential functions v_j and the nonrelativistic potential components \tilde{v}_j in (1) the spin representation the Fermi invariants is required:

$$\Omega_C = 1, \quad \Omega_{LS} = \frac{i}{2} (\vec{\sigma}^1 + \vec{\sigma}^2) \cdot \vec{n},$$

$$\begin{aligned}\Omega_T &= \vec{\sigma}^1 \cdot \vec{\sigma}^2 k^2 - 3\vec{\sigma}^1 \cdot \vec{k} \vec{\sigma}^2 \cdot \vec{k}, \quad \Omega_{SS} = \vec{\sigma}^1 \cdot \vec{\sigma}^2, \\ \Omega_{LS2} &= \vec{\sigma}^1 \cdot \vec{n} \vec{\sigma}^2 \cdot \vec{n}.\end{aligned}\tag{6}$$

Here the momenta \vec{k}, \vec{P} and \vec{n} are defined as $\vec{k} = \vec{p}' - \vec{p}$, $\vec{P} = (1/2)(\vec{p}' + \vec{p})$ and $\vec{n} = \vec{k} \times \vec{P}$, where \vec{p}' and \vec{p} are the final and initial relative momenta. The linear relation between the spin operators Ω_j and the Fermi invariants F_j is given (to order $1/m_N^2$) in ref. [4]. Since the nucleon mass $m_N \sim \mathcal{O}(N_C)$, only the leading terms in $1/m_N^2$ are needed for the present large N_C limit considerations.

The momentum space representation of the interaction (1) is

$$V_{NN} = \sum_i^5 [w_j^+ + w_j^- \vec{\tau}^1 \cdot \vec{\tau}^2] \Omega_j.\tag{7}$$

Note that $w_{LS2}\Omega_{LS2}$ only represents part of the complete Fourier transform of $\tilde{v}_{LS2}\tilde{\Omega}_{LS2}$, which also contains a linear combination of a term with the operators $\vec{\sigma}_1 \cdot \vec{P} \vec{\sigma}_2 \cdot \vec{P} - \vec{P}^2 \vec{\sigma}_1 \cdot \vec{\sigma}_2$, Ω_3 and Ω_4 [11]. These additional terms are of the same order in $1/N_C$ as the term $w_{LS2}^\pm \Omega_{LS2}$.

For a local potential one has explicitly [14]

$$\begin{aligned}w_C^\pm(k) &= 4\pi \int_0^\infty dr r^2 j_0(kr) \tilde{v}_C^\pm(r), \\ w_{LS}^\pm(k) &= -\frac{4\pi}{k} \int_0^\infty dr r^3 j_1(kr) \tilde{v}_{LS}^\pm(r), \\ w_T^\pm(k) &= \frac{4\pi}{k^2} \int_0^\infty dr r^2 j_2(kr) \tilde{v}_T^\pm(r), \\ w_{SS}^\pm(k) &= 4\pi \int_0^\infty dr r^2 j_0(kr) \tilde{v}_{SS}^\pm(r), \\ w_{LS2}^\pm(k) &= -\frac{4\pi}{k^2} \int_0^\infty dr r^4 j_2(kr) \tilde{v}_{LS2}^\pm(r).\end{aligned}\tag{8}$$

2.2 Order in N_C of the potential components

The N_C dependence of the components of the potential (7) may be inferred directly by quark model considerations. Consider the single quark operators $1, \sigma_j, \tau_k, \sigma_j \tau_k$. Matrix elements of the sum over N_C such quark operators in nucleon states then depend on N_C as follows [15]:

$$\begin{aligned}\langle N | \sum_{q=1}^{N_C} 1^q | N \rangle &\sim N_C, \quad \langle N | \sum_{q=1}^{N_C} \sigma_j^q | N \rangle \sim N_C^0 \\ \langle N | \sum_{q=1}^{N_C} \tau_k^q | N \rangle &\sim N_C^0, \quad \langle N | \sum_{q=1}^{N_C} \sigma_j^q \tau_k^q | N \rangle \sim N_C.\end{aligned}\tag{9}$$

In the large N_C limit the baryon-baryon interaction may be interpreted as meson exchange, since the gluon lines may be replaced by $q\bar{q}$ lines in all surviving (planar gluon) diagrams. The meson-baryon couplings are proportional to the quark operator matrix elements above and inversely proportional to the meson decay constants f_M , which scale like $\sqrt{N_C}$ [1]. Application of the scaling relations (9) and multiplication by $1/f_M^2$ to the two nucleon system directly implies that

$$\begin{aligned} w_C^+ \Omega_C, \quad w_T^- \Omega_T, \quad w_{SS}^- \Omega_{SS} &\sim \mathcal{O}(N_C), \\ w_C^- \Omega_C, \quad w_T^+ \Omega_T, \quad w_{SS}^+ \Omega_{SS} &\sim \mathcal{O}(1/N_C). \end{aligned} \quad (10)$$

The order in N_C of the spin-orbit interaction may be found as follows. The non-local momentum operator \vec{P} in the spin-orbit interaction operator Ω_{LS} always appears in the combination \vec{P}/m_N , where $m_N \sim N_C$. The isospin independent term $w_{LS}^+ \Omega_{LS}$ contains the spin-operator of one nucleon in combination with \vec{P}/m_N , and therefore the vertex scales as $1/N_C$. If the quark coupling at the other nucleon line is $\sim 1^q$, it follows that $w_{LS}^+ \Omega_{LS} \sim \mathcal{O}(1/N_C)$ once the overall factor $1/f_M^2$ is taken into account. For the isospin dependent spin-orbit interaction the scaling factor or the vertex with a spin-operator multiplying \vec{P}/m_N and an isospin operator is N_C^0 . In this case the vertex at the other nucleon line also contains an isospin factor, so it also scales as N_C^0 . Thus, it follows after multiplication with $1/f_M^2$ that $w_{LS}^- \Omega_{LS} \sim \mathcal{O}(1/N_C)$.

To find the N_C scaling of the isospin independent quadratic spin-orbit interaction one notes that there is a combination of a spin operator with \vec{P}/m_N at both nucleon lines, and thus the scaling factor is $1/N_C^2$, which after multiplication with $1/f_M^2$ gives rise to the scaling $w_{LS2}^+ \Omega_{LS2} \sim \mathcal{O}(1/N_C^3)$. The corresponding scaling factor for the isospin independent quadratic spin-orbit interaction is $w_{LS2}^- \Omega_{LS2} \sim \mathcal{O}(1/N_C)$, because the isospin operators at both nucleon lines bring an additional factor N_C each. This completes the derivation of the Manohar-Kaplan scaling relations (3) [2].

The linear relation between the Fermi invariant potential components v_j^\pm and the components w_j^\pm is given in ref. [4]. In the present application, only the leading terms in $1/m_N^2$ (i.e. in $1/N_C^2$) have to be retained in the transformation matrix.

To leading order in $1/N_C$ the Fermi invariant potential components v_j^\pm are then given as:

$$\begin{aligned} v_S^+ &= \frac{3}{4}w_C^+ - \frac{m_N^2}{2}w_{LS}^+, \\ v_S^- &= -\frac{m_N^2}{2}w_{LS}^- + \frac{k^2}{4}w_T^- + \frac{1}{4}w_{SS}^- + \frac{m_N^2 k^2}{4}w_{LS2}^-, \\ v_V^+ &= \frac{1}{4}w_C^+ + \frac{m_N^2}{2}w_{LS}^+, \\ v_V^- &= \frac{m_N^2}{2}w_{LS}^- - \frac{k^2}{4}w_T^- - \frac{1}{4}w_{SS}^- - \frac{m_N^2 k^2}{4}w_{LS2}^-, \\ v_T^+ &= \frac{k^2}{32m_N^2}w_C^+ + \frac{k^2}{16}w_{LS}^+ + \frac{k^2}{2}w_T^+ + \frac{1}{2}w_{SS}^+ + \frac{m_N^2 k^2}{4}w_{LS2}^+, \end{aligned}$$

$$\begin{aligned}
v_T^- &= \frac{k^2}{2} w_T^- + \frac{1}{2} w_{SS}^- + \frac{m_N^2 k^2}{4} w_{LS2}^-, \\
v_A^+ &= \frac{k^2}{32m_N^2} w_C^+ + \frac{k^2}{16} w_{LS}^+ + \frac{k^2}{2} w_T^+ + \frac{1}{2} w_{SS}^+ - \frac{m_N^2 k^2}{4} w_{LS2}^+, \\
v_A^- &= \frac{k^2}{2} w_T^- + \frac{1}{2} w_{SS}^- - \frac{m_N^2 k^2}{4} w_{LS2}^-, \\
v_P^+ &= \frac{1}{4} w_C^+ + \frac{m_N^2}{2} w_{LS}^+ + 12m_N^2 w_T^+, \\
v_P^- &= 12m_N^2 w_T^-.
\end{aligned} \tag{11}$$

From these expressions it follows that both the isospin independent and the isospin dependent scalar and vector potentials $v_1^\pm S$, $v_2^\pm V$ are $\sim \mathcal{O}(N_C)$. The isospin dependent tensor and axial vector potentials $v_T^- T$, $v_A^- A$ are $\sim \mathcal{O}(N_C)$, whereas the corresponding isospin independent interaction components $v_T^+ T$, $v_A^+ A$ are $\sim \mathcal{O}(1/N_C)$. The orders of the pseudoscalar potentials are $v_P^+ P \sim N_C$ and $v_P^- P \sim N_C^3$. All the coefficients in the expression of the pseudoscalar operator P in terms of the spin operators Ω_j are $\sim 1/m_N^2$ [4]. The corresponding potentials in the spin representation are therefore smaller by $1/N_C^2$ and as a consequence the highest order in N_C of the potentials components in the spin representation is $\mathcal{O}(N_C)$. This becomes explicit if the pseudoscalar invariant P is replaced by the on-shell equivalent pseudovector invariant P' :

$$P' = \frac{1}{4m_N^2} \gamma_\mu^1 k_\mu \gamma_5^1 \gamma_\nu^2 k_\nu \gamma_5^2, \tag{12}$$

which has the form required by spontaneous breaking of the approximate chiral symmetry of *QCD*. Because the pseudovector invariant builds in the required pair suppression in the pion-nucleon amplitude, and vanishes with the 4-momentum of the exchanged system, it is simpler to apply in relativistic approaches to nuclear reaction phenomenology [16]. These results are summarized in Table 1.

2.3 Dynamical interpretation of the potential components

The components of highest order in $1/N_C$ of the Fermi invariant potential components in Table 1 admit a direct dynamical interpretation in terms of meson exchange between the nucleons.

The only isospin independent potential components that are of order N_C (i.e. “large”) are the scalar and vector components v_S^+ and v_V^+ as well as the pseudoscalar component v_P^+ . The first one of these corresponds to the largest component in the two-pion exchange interaction between nucleons [17, 18], which commonly is modeled in terms of a strong scalar (“ σ ”) meson exchange mechanism [19, 20]. This potential component gives rise to an attractive interaction at intermediate range and the main agent for nuclear binding. The second corresponds to the short range repulsion

Isospin	S	V	T	A	P
1	N_C	N_C	$1/N_C$	$1/N_C$	N_C
$\vec{\tau}^1 \cdot \vec{\tau}^2$	N_C	N_C	N_C	N_C	N_C^3

Table 1: Order of the leading term in the $1/N_C$ expansion of the Fermi invariant potential components.

between nucleons, and is commonly modeled in terms of an ω meson exchange interaction, with an overstrength effective ω nucleon coupling constant [20]. The last (pseudoscalar) term, which may be interpreted as η meson exchange is large in the earlier potential models, but very small in the most recently developed models.

Among the isospin dependent Fermi invariant potential components in Table 1 the pseudoscalar component is of order N_C^3 and are thus the largest of all terms in the $1/N_C$ counting scheme. This strong pseudoscalar exchange component $v_{\vec{p}}$ is immediately interpretable in terms of the long range pion exchange interaction between nucleons, which is expectedly “strong”, more because of its long range than because of the strength of the pion-nucleon coupling.

That the other isospin dependent Fermi invariant potential components are of order N_C also corresponds to established nucleon-nucleon phenomenology. To see this, it is worth noting that an isospin 1 vector meson (ρ) exchange interaction may be expressed in terms of Fermi invariants as follows [14]:

$$\begin{aligned}
(\gamma_\mu^1 - \frac{\kappa}{2m_N} \sigma_{\mu\nu}^1 k_\nu)(\gamma_\mu^2 + \frac{\kappa}{2m_N} \sigma_{\mu\alpha}^2 k_\alpha) &= \frac{\kappa}{m_N^2} P_\mu^1 P_\mu^2 S + (1 + \kappa) V \\
&- \frac{\kappa(1 + \kappa)k^2}{4m_N^2} T + \frac{\kappa(1 + \kappa)}{m_N^2} P_\mu^1 P_\mu^2 P.
\end{aligned} \tag{13}$$

Here to leading order in $1/m_N^2$ (or $1/N_C^2$):

$$P_\mu^1 P_\mu^2 = -(m_N^2 + \frac{k^2}{4} + 2\vec{P}^2). \tag{14}$$

Phenomenological boson exchange interaction models typically contain a ρ meson exchange interaction, where the “effective” tensor coupling of the ρ meson to nuclei is large $\kappa \sim 7$ [20] even

though the vector coupling is small. Because of the large tensor coupling such an isospin dependent vector meson exchange interaction contributes strongly to all the Fermi invariant potential components, except the axial vector invariant. The order N_C of the potential components v_S^-, v_V^-, v_T^- thus may be interpreted in terms of a strong ρ meson exchange (or ρ exchange-like) interaction. The isospin independent vector meson (ω) exchange in contrast only contributes strongly to the vector V invariant potential, because of the small ω -nucleon tensor coupling.

While the isospin dependent pseudoscalar potential $v_P^- P$ is of order N_C^3 the corresponding isospin independent pseudoscalar exchange potential $v_P^+ P$ is only of order N_C . This corresponds to the phenomenological finding in boson exchange models for the nucleon-nucleon interaction that the η meson exchange term is mostly much weaker than the π meson exchange interaction component.

3 Large N_C components of phenomenological interaction models

3.1 The scalar potential

For the visualization of the Fermi invariant potential components it is useful to plot the potential components in configuration space. The expression for the isospin independent and isospin dependent scalar potentials in configuration space are obtained from (11) as

$$\begin{aligned} v_S^+(r) &= \frac{3}{4} \tilde{v}_C^+(r) + \frac{m_N^2}{2} \int_r^\infty dr' r' \tilde{v}_{LS}^+(r'), \\ v_S^-(r) &= \frac{m_N^2}{2} \int_r^\infty dr' r' \tilde{v}_{LS}^-(r') - \frac{1}{4} \{ \tilde{v}_T^-(r) - 3 \int_r^\infty dr' \frac{\tilde{v}_T^-(r')}{r'} \} + \frac{1}{4} \tilde{v}_{SS}^-(r) \\ &\quad - \frac{m_N^2}{4} \{ r^2 \tilde{v}_{LS2}^-(r) - 3 \int_r^\infty dr' r' \tilde{v}_{LS2}^-(r') \}, \end{aligned} \quad (15)$$

Here, and below, only terms of leading order in $1/N_C$ have been retained, so that all terms on the r.h.s. are of the same order in $1/N_C$. These potential components as obtained from the phenomenological *V18* [9], the *CD - Bonn* [10], the *Nijmegen(93)* [11] and the *Paris* [12] potentials are shown in Figs. 1 and 2.

In the case of the *V18* potential, the operator form of which contains a quadratic spin-orbit interaction of the form $(\vec{L} \cdot \vec{S})^2$, we employ the relation

$$(\vec{L} \cdot \vec{S})^2 = 2\vec{L}^2 + 2\tilde{\Omega}_{LS2} - 2\tilde{\Omega}_{LS} \quad (16)$$

to reduce the interaction to standard form (1) in combination with terms of the form \vec{L}^2 . As the order of the remaining terms \vec{L}^2 , $(\vec{\tau}^1 \cdot \vec{\tau}^2) \vec{L}^2$, $(\vec{\sigma}^1 \cdot \vec{\sigma}^2) \vec{L}^2$ and $(\vec{\tau}^1 \cdot \vec{\tau}^2)(\vec{\sigma}^1 \cdot \vec{\sigma}^2) \vec{L}^2$ in the *V18* potential

may, by the argument in section (2.2), be shown to be at most of order $1/N_C^{-1}$, $1/N_C^{-3}$, $1/N_C^{-3}$ and $1/N_C^{-1}$, respectively (once multiplied by $1/f_M^2$), we shall not have to retain them in the leading order analysis here. The spin-orbit ($\tilde{\Omega}_{LS}$) and quadratic spin-orbit ($\tilde{\Omega}_{LS^2}$) interaction terms in the $V18$ interaction shall however be included with the corresponding terms in the expressions (15).

The $CD - Bonn$ interaction model is a single boson exchange model that is expressed in terms of Fermi invariants and in the case of vector meson exchange in the form, which is reducible to Fermi invariants by eqn. (13). Both the *Nijmegen*(93) and the (parametrized) *Paris* interaction models are expressed in the standard form (1). The central components of the *Paris* potential do in addition contain velocity dependent terms with the operator form \vec{P}^2 . As these, by the argument in section (2.2) are down by two orders in $1/N_C$ in comparison to the local terms, they are not considered here.

In Table 2 are listed the volume integrals of the the leading terms in the $1/N_C$ expansion of the Fermi invariant potential components of the phenomenological interaction models. In the case of the isospin independent scalar interaction these volume integrals are all of the order 10 fm^2 , ranging from -6.3 fm^2 for the *Paris* potential to -13.5 for the $CD - Bonn$ potential.

If the isospin independent scalar potential component is interpreted as due to a single scalar meson exchange interaction, the volume integral would equal $-g_S^2/m_S^2$, where g_S is the scalar meson – nucleon coupling constant, and m_S the scalar meson mass. The mass of the lowest scalar meson (σ or $f_0(400 - 1200)$) is not well established [21], but it is typically taken to be of the order $550 - 750 \text{ MeV}$ in phenomenological nucleon-nucleon interaction models [20]. If m_S is taken to be 600 MeV , within this range, the value of the effective scalar meson coupling constant for these interactions range between 7.6 (*Paris*) and 11.2 ($CD - Bonn$) (Table 3). These “effective” scalar meson coupling constant values should be viewed as lower limits, as any short range form factors multiplying the scalar meson exchange interaction would increase these values. It is striking how closely similar values obtain for the “effective” scalar meson coupling constants with the 4 different interaction models, two of which do not contain any scalar meson like term in their parametrized forms.

The isospin independent scalar interaction components of the 4 interaction models are rather similar for nucleon separations beyond 0.5 fm . The radial shapes differ considerably at short distances, however, ranging from attractive to repulsive. The two boson exchange models (*Nijmegen*(93) and $CD - Bonn$) are repulsive by construction. The more phenomenological $V18$ and *Paris* interaction models are repulsive at short distance, a feature which cannot be interpreted in terms of meson exchange, and which otherwise only is known from the Skyrme model [6].

The isospin dependent scalar interaction components of the 4 phenomenological interaction models are shown in Fig.2. These interaction components are of order $1/N_C$ and should therefore be weaker by an order of magnitude than the isospin independent scalar interaction components. This is in fact also revealed by comparison of Figs. 1 and 2. The volume integrals of these

interaction components range between -3.2 and -4.4 fm^2 , as shown in Table 2, and are thus about a factor 3-4 smaller than those of the corresponding isospin independent interactions.

In a meson exchange interpretation the isospin dependent scalar interaction arises from exchange of scalar mesons with isospin 1. The lightest of these is the $a_0(980)$. In a simple meson exchange model the volume integrals of these interactions should equal $-g_{a_0}^2/m_{a_0}^2$, where g_{a_0} is the $a_0(980)$ -nucleon coupling constant and m_{a_0} is the mass of the $a_0(980)$. If the value of the “effective” $a_0(980)$ nucleon coupling constant is extracted from these volume integrals, the values range from $g_{a_0} = 9.0$ to $g_{a_0} = 10.4$ (Table 3). In this case all the interaction models give very similar values for this coupling constant.

3.2 The vector potential

The expression for the isospin independent and isospin dependent vector potentials in configuration space are obtained from (11) as

$$\begin{aligned} v_V^+(r) &= \frac{1}{4} \tilde{v}_C^+(r) - \frac{m_N^2}{2} \int_r^\infty dr' r' \tilde{v}_{LS}^+(r'), \\ v_V^-(r) &= -\frac{m_N^2}{2} \int_r^\infty dr' r' \tilde{v}_{LS}^-(r') + \frac{1}{4} \{ \tilde{v}_T^-(r) - 3 \int_r^\infty dr' \frac{\tilde{v}_T^-(r')}{r'} \} - \frac{1}{4} \tilde{v}_{SS}^-(r) \\ &\quad + \frac{m_N^2}{4} \{ r^2 \tilde{v}_{LS2}^-(r) - 3 \int_r^\infty dr' r' \tilde{v}_{LS2}^-(r') \}, \end{aligned} \quad (17)$$

These potential components as obtained from the phenomenological *V18* [9], the *CD-Bonn* [10], the *Nijmegen(93)* [11] and the *Paris* [12] potentials are shown in Figs. 3 and 4. Note that in the large N_C limit $v_V^-(r) = -v_S^-(r)$. In a boson exchange interpretation this implies equality in strength between the $a_0(980)$ and $\rho(770)$ exchange interactions.

The isospin independent vector interaction admits an obvious interpretation in terms of a strongly repulsive ω meson exchange interaction. The shape of this interaction component is very similar for all the interactions considered, and the volume integrals are also remarkably similar, ranging from 8.7 (*Nijmegen(93)*) to 11.6 fm^2 (*CD-Bonn*). If the “effective” ω -nucleon coupling constant is extracted from the volume integrals, by setting them equal to g_ω^2/m_ω^2 the values range between 11.7 and 13.5 . The similarity between these coupling constant values is particularly notable in the case of the *V18* and the parametrized *Paris* interaction models, as neither one of these contains any explicit vector meson exchange term.

The isospin dependent vector interaction, by eqn. (13), admits an interpretation in terms of ρ -meson exchange. This interaction component is roughly 3 times weaker than the corresponding isospin independent interaction. The volume integrals of the isospin dependent vector interaction range from 2.9 to 4.4 fm^2 for the 4 phenomenological interactions considered here. In a simple

Component	$V18$	$CD - Bonn$	$Nijmegen(93)$	$Paris$	$\mathcal{O}(N_C)$
v_S^+	-8.7	-13.5	-10.3	-6.3	N_C
v_S^-	-3.2	-3.3	-3.3	-4.4	N_C
v_V^+	9.4	11.6	8.7	10.2	N_C
v_V^-	3.2	3.2	2.9	4.4	N_C
v_T^+	-0.1	0.0	0.0001	0.03	$1/N_C$
v_T^-	0.6	0.1	0.001	0.46	N_C
v_A^+	-0.1	0	0	0.03	$1/N_C$
v_A^-	0.6	0	0	0.46	N_C
v_P^+	9.8	0.0	0.35	18.0	N_C
v_P^-	360	338	323	352	N_C^3

Table 2: Volume integrals (in fm²) of the leading terms in the $1/N_C$ expansion of the Fermi invariant potential components phenomenological interaction models.

Component	<i>V18</i>	<i>CD – Bonn</i>	<i>Nijmegen(93)</i>	<i>Paris</i>
g_σ	9.0	11.2	9.8	7.6
g_{a_0}	9.0	9.0	9.0	10.4
g_ω	12.2	13.5	11.7	12.7
κ_ρ	7.0	7.0	6.3	10.1
g_η	8.7	0.0	1.8	11.7
g_π	13.4	13.0	12.7	13.2

Table 3: Effective meson-nucleon coupling values that correspond to phenomenological interaction models.

ρ -meson exchange model these volume integrals have the form $(1 + \kappa)g_\rho^2/m_\rho^2$, where g_ρ and κ are the vector and tensor ρ -meson-nucleon coupling constants. The canonical value for the ρ -nucleon vector coupling constant is $g_\rho^2/4\pi \sim 0.5$. If this value for g_ρ is used in the expression for the volume integral, an “effective” value for κ_ρ can be determined. The volume integrals in Table 2 for the 4 interaction models considered then give values for κ_ρ in the range 6.3 – 10.1. These values support the early finding that $\kappa_\rho \sim 6.6$ [22]. Both the *Nijmegen93* and the *CD – Bonn* interaction use the somewhat larger value $g_\rho^2/4\pi = 0.84$ for the ρ -nucleon coupling constant. This larger value would reduce the values for κ_ρ in Table 3 by about 40 %.

From eqns. (15) and (17) it follows that in the limit of large N_C the isospin dependent scalar and vector exchange potentials should be equal in strength and of opposite sign. If imposed on a boson exchange model this condition implies that

$$g_{a_0}^2 = (1 + \kappa_\rho)g_\rho^2. \quad (18)$$

Comparison of the form and strength of these potential components for the *CD – Bonn* and *Nijmegen93* potential models in Figs. 2 and 4 shows that these potential models satisfy this constraint very well.

3.3 The tensor potential

The expression for the isospin independent and isospin dependent tensor potentials in configuration space are obtained from (11) as

$$\begin{aligned} v_T^+(r) &= -\frac{1}{32m_N^2}\{\tilde{v}_C^{+''}(r) + \frac{2}{r}\tilde{v}_C^{+'}(r)\} + \frac{1}{16}\{3\tilde{v}_{LS}^+(r) + r\tilde{v}_{LS}^{+'}(r)\} \\ &\quad -\frac{1}{2}\{\tilde{v}_T^+(r) - 3\int_r^\infty dr' \frac{\tilde{v}_T^+(r')}{r'}\} + \frac{1}{2}\tilde{v}_{SS}^+(r) - \frac{m_N^2}{4}\{r^2\tilde{v}_{LS2}^+(r) - 3\int_r^\infty dr'r'\tilde{v}_{LS2}^+(r')\}, \\ v_T^-(r) &= -\frac{1}{2}\{\tilde{v}_T^-(r) - 3\int_r^\infty dr' \frac{\tilde{v}_T^-(r')}{r'}\} + \frac{1}{2}\tilde{v}_{SS}^-(r) \\ &\quad -\frac{m_N^2}{4}\{r^2\tilde{v}_{LS2}^-(r) - 3\int_r^\infty dr'r'\tilde{v}_{LS2}^-(r')\}, \end{aligned} \quad (19)$$

These potential components as obtained from the phenomenological *V18* [9], the *CD – Bonn* [10], the *Nijmegen93* [11] and the *Paris* [12] potentials are shown in Figs. 5 and 6.

In a meson exchange model the tensor potential arises from vector meson exchange when the vector mesons couple to the nucleon with a Pauli (tensor) coupling. As in the *CD – Bonn* interaction model the isospin independent ω meson exchange interaction has no Pauli coupling term v_T^+ vanishes in this interaction model.

By the N_C counting rules in Table 1 the isospin independent tensor interaction should be smaller by $1/N_C^2$ than the isospin dependent tensor interaction. Comparison of the corresponding set of volume integrals in Table 2 shows that the phenomenological interaction models considered here satisfy this rule well.

The order N_C of the isospin dependent tensor interaction alone does not explain why this interaction is one order of magnitude smaller than the corresponding vector interactions for all the phenomenological potential models. For boson exchange models the reason is readily seen in eqn. (13), which shows that the tensor coupled vector meson exchange interaction is suppressed by an overall factor $1/m_N^2$, which is only partially counteracted by the large tensor coupling κ_ρ . For the phenomenological interaction models the reason for the weakness of the isospin dependent tensor interaction is to be found in the fact that pion exchange gives rise to the bulk of the isospin dependent spin-spin and vector interactions, and this pion (or pseudoscalar) exchange contribution is exactly cancelled in the combination of v_{SS}^- and v_T^- in eqn. (19). This latter argument may also be extended to the isospin independent tensor interaction, where the pseudoscalar - in this case the η meson - exchange contribution cancels in the combination of v_{SS}^+ and v_T^+ in (19).

3.4 The axial vector potential

The expression for the isospin independent and isospin dependent axial vector potentials in configuration space are obtained from (11) as

$$\begin{aligned}
v_A^+(r) &= -\frac{1}{32m_N^2}\{\tilde{v}_C^{+''}(r) + \frac{2}{r}\tilde{v}_C^{+'}(r)\} + \frac{1}{16}\{3\tilde{v}_{LS}^+(r) + r\tilde{v}_{LS}^{+'}(r)\} \\
&\quad -\frac{1}{2}\{\tilde{v}_T^+(r) - 3\int_r^\infty dr' \frac{\tilde{v}_T^+(r')}{r'}\} + \frac{1}{2}\tilde{v}_{SS}^+(r) + \frac{m_N^2}{4}\{r^2\tilde{v}_{LS2}^+(r) - 3\int_r^\infty dr' r' \tilde{v}_{LS2}^+(r')\}, \\
v_A^-(r) &= -\frac{1}{2}\{\tilde{v}_T^-(r) - 3\int_r^\infty dr' \frac{\tilde{v}_T^-(r')}{r'}\} + \frac{1}{2}\tilde{v}_{SS}^-(r) \\
&\quad + \frac{m_N^2}{4}\{r^2\tilde{v}_{LS2}^-(r) - 3\int_r^\infty dr' r' \tilde{v}_{LS2}^-(r')\},
\end{aligned} \tag{20}$$

These potential components as obtained from the phenomenological *V18* [9], and the *Paris* [12] potentials are shown in Figs. 7 and 8. In a single meson exchange model only axial vector meson exchange gives rise to an axial vector interaction. Since the *CD - Bonn* and the *Nijmegen93* boson exchange interaction models do not contain any a_1 meson exchange terms these interaction models do not have any axial vector exchange components. The phenomenological *V18* and *Paris* potential models give rise to small A interaction components. These satisfy the N_C counting rules in Table 1, by which the isospin dependent axial vector interaction should be larger by N_C^2 than the isospin independent one as may be seen from the volume integrals in Table 2. The substantial

strength of the axial vector exchange interaction components of the *Paris* potential model are likely to be artifacts of the parametrization rather than genuine consequences of the NN scattering data.

3.5 The pseudoscalar potential

The expression for the isospin independent and isospin dependent pseudoscalar potentials in configuration space are obtained from (11) as

$$\begin{aligned} v_P^+(r) &= \frac{1}{4} \tilde{v}_C^+(r) - \frac{m_N^2}{2} \int_r^\infty dr' r' \tilde{v}_{LS}^+(r') + 12m_N^2 \int_r^\infty dr' r' \int_{r'}^\infty dr'' \frac{\tilde{v}_T^+(r'')}{r''} \\ v_P^-(r) &= 12m_N^2 \int_r^\infty dr' r' \int_{r'}^\infty dr'' \frac{\tilde{v}_T^-(r'')}{r''}. \end{aligned} \quad (21)$$

These potential components as obtained from the phenomenological $V18$ [9], the $CD - Bonn$ and the *Nijmegen*(93) [11] and the *Paris* [12] potentials are shown in Figs. 9 and 10.

In a single meson exchange interpretation the longest range mechanism, that contributes to the isospin independent pseudoscalar interaction, is η meson exchange. This interaction component is not well constrained by nucleon-nucleon scattering data. As a consequence the phenomenological interaction models considered here give widely different results for this interaction component, as is evident in Fig. 9.

The η -nucleon coupling constant is also not well known. An estimate for this coupling constant may be obtained from the volume integrals of the phenomenological interaction models if these are set to equal g_η^2/m_η^2 . From the volume integral values listed in Table 2 one then obtains values for g_η , which range from 1.8 (*Nijmegen*(93)) to 11.7 (*Paris*) (Table 3). Analyses of observables, other than nucleon-nucleon scattering, as e.g. η -meson photoproduction suggest that the coupling constant value should not exceed 2.2 [23].

The isospin dependent pseudoscalar exchange interaction is strong and has long range. Its main component is the long range pion exchange interaction, which is built into all the phenomenological interaction models considered here. Because of this all the interaction models converge for nucleon separations larger than 1 fm. This interaction component is also the strongest component by N_C counting, as it scales as N_C^3 (Table 1). The volume integrals of this interaction component are listed in Table 2 for the 4 interaction models considered, and range from 323 fm² to 360 fm². The corresponding values for the pseudoscalar pion-nucleon coupling constant g_π range from 12.7 to 13.4, when extracted by equating the numerically determined volume integrals with g_π^2/m_π^2 (Table 3).

4 Electromagnetic exchange current in the large N_C limit

The isoscalar and isovector components of the single nucleon charge operator are of order N_C^1 and N_C^0 respectively, as may be inferred from the scaling relations (9). Because of their inverse dependence on the nucleon mass the isoscalar and isovector convection current components of the single nucleon current scale as N_C^0 and N_C^{-1} in comparison. In the case of the spin component of the single nucleon current the isoscalar component scales in contrast as N_C^{-1} , whereas the N_C dependence of the isovector part is N_C^0 .

Associated with the isospin dependent interactions expressed in terms of Fermi invariants are corresponding two-nucleon electromagnetic interaction currents with the isospin dependence $(\vec{\tau}^1 \times \vec{\tau}^2)_3$ [4]. In a meson exchange interpretation, these interaction currents correspond to the currents carried by exchanged charged mesons (Fig. 11b). These current operators are listed in Table 4, along with the corresponding N_C scaling power. The N_C scaling factors of these current operators may be derived by reduction to the spin representation, and application of the relations (9). In the expressions in Table 4 the potential functions $v_j^\pm(k)$ represent the Fermi invariant potential components in momentum space. The N_C dependence of these interaction currents should be compared to the N_C scaling of the corresponding components of the single nucleon charge and current operators.

The charge components of the interaction currents in Table 4 are suppressed by their dependence on the energy exchanged between the nucleons. This is proportional to the nucleon momenta and inversely proportional to the nucleon mass ($\sim 1/N_C$). In a frame with no energy transfer they vanish. In the reduction to the spin representation exchange charge operators also arise in the elimination of negative energy intermediate states (“pair terms”). These are listed in Table 5. These operators have an overall factor $1/m_N^3$, which arises from the small components of the Dirac spinors and the propagator of the intermediate negative energy propagator (Fig. 11a). The order in $1/N_C$ of these operators follow from the general scaling rules (9) and the overall factor $1/m_N^3 \sim 1/N_C^3$. In Table 5 and in subsequent tables, the momenta of the two nucleons are denoted by $\vec{P}_j = 1/2(\vec{p}_j' + \vec{p}_j)$ with $j = 1, 2$ respectively.

The largest N_C scaling factors of the exchange charge operators that are associated with the 5 Fermi invariant components of the interaction are also listed in Table 5, separately for the isospin independent and isospin dependent interaction components. The order of the exchange charge operators are smaller by $1/N_C^3$ in comparison with the corresponding single nucleon charge operators. The terms with the highest N_C dependence are those, which are independent of spin and isospin and those, which contain the spin-isospin bilinears $\sigma_i \tau_j$ of both nucleons. The remaining terms are smaller by at least one power of $1/N_C$.

The interaction currents that are listed in Table 4 arise from the coupling of the electromagnetic field to the charge exchanged between the nucleons (Fig. 11b). Their explicit expressions in the spin representation have been given in ref. [4]. The components of these currents, which are of

Fermi invariant	j_μ	N_C
S	$i(\vec{\tau}^1 \times \vec{\tau}^2)_3 \frac{v_S^-(k_2) - v_S^-(k_1)}{k_2^2 - k_1^2} (k_{2\mu} - k_{1\mu}) S$	N_C^{-1}
V	$\{i(\vec{\tau}^1 \times \vec{\tau}^2)_3 \frac{v_V^-(k_2) - v_V^-(k_1)}{k_2^2 - k_1^2} [(k_{2\mu} - k_{1\mu})V + \gamma_\mu^1(\gamma^2 \cdot k_1) - \gamma_\mu^2(\gamma^1 \cdot k_2)]\}$	N_C^{-1}
T	$\{i(\vec{\tau}^1 \times \vec{\tau}^2)_3 \frac{v_T^-(k_2) - v_T^-(k_1)}{k_2^2 - k_1^2} [(k_{2\mu} - k_{1\mu})T + k_{2\nu}\sigma_{\mu\alpha}^2\sigma_{\alpha\nu}^1 - k_{1\nu}\sigma_{\mu\alpha}^1\sigma_{\alpha\nu}^2]\}$	N_C^1
A	$i(\vec{\tau}^1 \times \vec{\tau}^2)_3 \frac{v_A^-(k_2) - v_A^-(k_1)}{k_2^2 - k_1^2} (k_{2\mu} - k_{1\mu}) A$	N_C^1
P	$i(\vec{\tau}^1 \times \vec{\tau}^2)_3 \frac{v_P^-(k_2) - v_P^-(k_1)}{k_2^2 - k_1^2} (k_{2\mu} - k_{1\mu}) P$	N_C^{-1}

Table 4: Interaction currents and N_C scaling factors associated with the interaction components. The fractions of the imparted momentum q to the two nucleons are denoted k_1 and k_2 . Note that in the case of the A invariant a conserved current obtains only in combination with a P invariant term [4].

Fermi invariant	ρ	v_j^+	v_j^-
S	$\frac{1}{4m_N^3}[\vec{q}^2 + 2i\vec{\sigma}^1 \cdot \vec{P}_1 \times \vec{q}]$ $[v_S^+(k_2)\hat{e}_1 + v_S^-(k_2)\tilde{e}_2] + (1 \leftrightarrow 2)$	N_C^{-2}	N_C^{-3}
V	$\frac{1}{4m_N^3}[\vec{q} \cdot \vec{k}_2 + (\vec{\sigma}^2 \times \vec{k}_2) \cdot (\vec{\sigma}^1 \times \vec{q}) - 2i\vec{\sigma}^1 \cdot \vec{q} \times \vec{P}_2]$ $[v_V^+(k_2)\hat{e}_1 + v_V^-(k_2)\tilde{e}_2] + (1 \leftrightarrow 2)$	N_C^{-2}	N_C^{-2}
T	$\frac{1}{4m_N^3}[\vec{\sigma}^1 \cdot \vec{\sigma}^2 \vec{q} \cdot \vec{k}_2 - \vec{\sigma}^2 \cdot \vec{q} \vec{\sigma}^1 \cdot (\vec{k}_2 - \vec{q})$ $+ \vec{q} \cdot \vec{k}_2 - 2i\vec{\sigma}^2 \cdot \vec{q} \times \vec{P}_2]$ $[v_T^+(k_2)\hat{e}_1 + v_T^-(k_2)\tilde{e}_2] + (1 \leftrightarrow 2)$	N_C^{-2}	N_C^{-2}
A	$\frac{1}{4m_N^3}[\vec{q}^2 \vec{\sigma}^1 \cdot \vec{\sigma}^2 + 2i\vec{\sigma}^2 \cdot \vec{P}_1 \times \vec{q}$ $- \vec{\sigma}^1 \cdot \vec{q} \vec{\sigma}^2 \cdot \vec{q} + \vec{\sigma}^1 \cdot \vec{q} \vec{\sigma}^2 \cdot \vec{k}_2]$ $[v_A^+(k_2)\hat{e}_1 + v_A^-(k_2)\tilde{e}_2] + (1 \leftrightarrow 2)$	N_C^{-2}	N_C^{-2}
P	$\frac{1}{4m_N^3}\vec{\sigma}^2 \cdot \vec{k}_2 \vec{\sigma}^1 \cdot \vec{q}$ $[v_P^+(k_2)\hat{e}_1 + v_P^-(k_2)\tilde{e}_2] + (1 \leftrightarrow 2)$	N_C^{-3}	$N_C^{-2},$

Table 5: Exchange charge operators and the highest N_C scaling factors associated with the interaction components. The fractions of the imparted momentum q to the two nucleons are denoted k_1 and k_2 respectively. The isospin factors are defined as $\hat{e}_1 = (1 + \tau_3^1)/2$ and $\tilde{e}_2 = (\vec{\tau}^1 \cdot \vec{\tau}^2 + \tau_3^2)/2$.

highest order in N_C are listed in Table 6.

The N_C dependence of these internal coupling interaction currents that are associated with the different Fermi invariant components of the phenomenological interaction model is smaller by one power of $1/N_C$ than that of the isovector part of the single nucleon current, with exception for the currents that are associated with the T and A invariants, which anomalously scale as N_C^1 . As these interaction components are very weak in all the phenomenological models considered, and are suppressed by $1/m_N^2 \sim 1/N_C^2$ in meson exchange models, it is natural to conclude that their N_C dependence in practice also is N_C^{-1} rather than N_C^1 .

In the reduction to the spin representation the negative energy pole terms give rise to seagull type exchange currents, in which the electromagnetic field couples to either one of the interaction nucleons with a point interaction to the interaction (Fig. 11a). The exchange currents of this type, which have the highest N_C scaling factors (in this case $1/N_C$) are listed in Table 7. The $1/N_C$ dependence of these current operators are similar to the internal charge coupling currents in Table 6.

5 Axial exchange current in the large N_C limit

The axial current operator of a single nucleon,

$$\vec{A}_\pm = -g_A \vec{\sigma} \tau_\pm, \quad (22)$$

is of order N_C^1 (cf. (9)). The axial vector interaction currents that are associated with the Fermi invariant components of the nucleon-nucleon interaction have been listed in ref. [25]. These currents only arise in the reduction to the spin representation, which brings in an overall factor $1/M_N^3$. The $1/N_C$ scaling factors of these axial exchange current operators may be determined from their operator structure by means of the relations (9). It follows from their vector nature that they are $1/N_C^3$ smaller than those of the corresponding interaction components.

The subset of axial exchange currents that are associated with the Fermi invariant interaction components, and which are of order $1/N_C^2$ are listed in Table 8. In the case of the pseudoscalar invariant the reduction to the spin representation brings an overall factor $1/M_N^5$. The axial exchange current that is associated with the pseudoscalar invariant is therefore of the order N_C^{-5} .

The axial exchange charge operators that are associated with Fermi invariant decomposition of the nucleon-nucleon interaction have been derived in ref. [3], and are listed in Table 9. For these operators the reduction to the spin representation only brings in an overall factor of $1/m_N^2$. The $1/N_C$ scaling factors of these operators are accordingly smaller by a factor $1/N_C^2$ than those of the corresponding interaction components.

Fermi invariant	\vec{j}	$\mathcal{O}(N_C)$
S	$i(\vec{\tau}^1 \times \vec{\tau}^2)_3 \frac{v_S^-(k_2) - v_S^-(k_1)}{k_2^2 - k_1^2} (\vec{k}_2 - \vec{k}_1)$	N_C^{-1}
V	$\{i(\vec{\tau}^1 \times \vec{\tau}^2)_3 \frac{v_V^-(k_2) - v_V^-(k_1)}{k_2^2 - k_1^2}$ $\{(\vec{k}_2 - \vec{k}_1) \{1 + \frac{1}{4m_N^2} [\vec{\sigma}^1 \cdot \vec{\sigma}^2 \vec{k}_1 \cdot \vec{k}_2 - \vec{\sigma}^1 \cdot \vec{k}_2 \vec{\sigma}^2 \cdot \vec{k}_1]$ $- \frac{1}{4m_N^2} [\vec{\sigma}^1 \times \vec{k}_1 \vec{\sigma}^2 \cdot \vec{k}_1 \times \vec{k}_2 + \vec{\sigma}^2 \times \vec{k}_2 \vec{\sigma}^1 \cdot \vec{k}_1 \times \vec{k}_2]\}$	N_C^{-1}
T	$\{i(\vec{\tau}^1 \times \vec{\tau}^2)_3 \frac{v_T^-(k_2) - v_T^-(k_1)}{k_2^2 - k_1^2} [\vec{\sigma}^1 \vec{\sigma}^2 \cdot \vec{k}_2 - \vec{\sigma}^2 \vec{\sigma}^1 \cdot \vec{k}_1]$	N_C^1
A	$i(\vec{\tau}^1 \times \vec{\tau}^2)_3 \frac{v_A^-(k_2) - v_A^-(k_1)}{k_2^2 - k_1^2} (\vec{k}_2 - \vec{k}_1) \vec{\sigma}^1 \cdot \vec{\sigma}^2$	N_C^1
P	$\frac{i}{4m_N^2} (\vec{\tau}^1 \times \vec{\tau}^2)_3 \frac{v_P^-(k_2) - v_P^-(k_1)}{k_2^2 - k_1^2} (\vec{k}_2 - \vec{k}_1) \vec{\sigma}^1 \cdot \vec{k}_1 \vec{\sigma}^2 \cdot \vec{k}_2$	N_C^{-1}

Table 6: Interaction currents that are associated with the Fermi invariant interaction components that arise from e.m. coupling to the exchanged charge, which have the highest N_C scaling factors associated with the interaction components. The fractions of the imparted momentum q to the two nucleons are denoted k_1 and k_2 respectively.

Fermi invariant	\vec{j}	$\mathcal{O}(N_C)$
S	$-\frac{1}{4m_N^2}(1 + \vec{\tau}_3^1)v_S^+(k_2)(i\vec{\sigma}^1 \times \vec{q} + 2\vec{P}_1) + (1 \leftrightarrow 2)$	N_C^{-1}
V	$-\frac{1}{4m_N^2}\{(1 + \vec{\tau}_3^1)v_V^+(k_2)(i(\vec{\sigma}^1 + \vec{\sigma}^2) \times \vec{k}_2 + 2\vec{P}_2)$ $+i(\vec{\tau}^1 \times \vec{\tau}^2)_3v_V^-(k_2)[\vec{k}_2 \vec{\sigma}^1 \cdot \vec{\sigma}^2 - \vec{\sigma}^2 \vec{\sigma}^1 \cdot \vec{k}_2]\} + (1 \leftrightarrow 2)$	N_C^{-1}
T	$-\frac{1}{4m_N^2}\{i(1 + \vec{\tau}_3^1)v_T^+(k_2)(\vec{\sigma}^1 + \vec{\sigma}^2) \times \vec{k}_2$ $+(\vec{\tau}^1 \cdot \vec{\tau}^2 + \vec{\tau}_3^2)v_T^-(k_2)[2\vec{\sigma}^1 \vec{\sigma}^2 \cdot \vec{P}_1 - 2\vec{\sigma}^2 \vec{\sigma}^1 \cdot \vec{P}_2 + 2\vec{P}_2 \vec{\sigma}^1 \cdot \vec{\sigma}^2]$ $+i(\vec{\tau}^1 \times \vec{\tau}^2)_3v_T^-(k_2)[\vec{\sigma}^1 \vec{\sigma}^2 \cdot \vec{q} + \vec{k}_2 \vec{\sigma}^1 \cdot \vec{\sigma}^2 - \vec{\sigma}^2 \vec{\sigma}^1 \cdot \vec{k}_2]\} + (1 \leftrightarrow 2)$	N_C^{-1}
A	$-\frac{1}{4m_N^2}\{(\vec{\tau}^1 \cdot \vec{\tau}^2 + \vec{\tau}_3^2)v_A^-(k_2)[2\vec{\sigma}^1 \vec{\sigma}^2 \cdot \vec{P}_2 - 2\vec{\sigma}^2 \vec{\sigma}^1 \cdot \vec{P}_1 + 2\vec{P}_1 \vec{\sigma}^1 \cdot \vec{\sigma}^2]$ $+i(\vec{\tau}^1 \times \vec{\tau}^2)_3v_A^-(k_2)[\vec{q} \vec{\sigma}^1 \cdot \vec{\sigma}^2 - \vec{\sigma}^2 \vec{\sigma}^1 \cdot \vec{q} + \vec{\sigma}^1 \vec{\sigma}^2 \cdot \vec{k}_2]\} + (1 \leftrightarrow 2)$	N_C^{-1}
P	$-\frac{i}{4m_N^2}(\vec{\tau}^1 \times \vec{\tau}^2)_3v_P^-(k_2)\vec{\sigma}^1 \vec{\sigma}^2 \cdot \vec{k}_2 + (1 \leftrightarrow 2)$	N_C^{-1}

Table 7: Interaction currents that arise from coupling to intermediate negative energy pole terms that have the largest N_C scaling factors. The fractions of the imparted momentum q to the two nucleons are denoted k_1 and k_2 respectively.

Fermi invariant	\vec{A}	$\mathcal{O}(N_C)$
S	$-\frac{g_A}{4m_N^3}v_S^+(k_2)\tau_{\pm}^1(4\vec{\sigma}^1\vec{P}_1^2 - 4\vec{P}_1\vec{\sigma}^1\cdot\vec{P}_1) + (1 \leftrightarrow 2)$	$1/N_C^2$
V	$-\frac{g_A}{4m_N^3}\{v_V^+(k_2)\tau_{\pm}^1[\frac{3}{4}\vec{\sigma}^1\vec{k}_2^2 - \vec{\sigma}^1\vec{P}_1^2 - \vec{k}_2\vec{\sigma}^1\cdot\vec{k}_2$ $+4\vec{\sigma}^1\vec{P}_1\cdot\vec{P}_2 - 4\vec{P}_2\vec{\sigma}^1\cdot\vec{P}_1] + v_V^-(k_2)\tau_{\pm}^2[\vec{\sigma}^2\vec{k}_2^2 - \vec{k}_2\vec{\sigma}^2\cdot\vec{k}_2]$ $+v_V^-(k_2)(\vec{\tau}^1\times\vec{\tau}^2)_{\pm}\vec{\sigma}^2\times\vec{k}_2\vec{\sigma}^1\cdot\vec{k}_2\} + (1 \leftrightarrow 2)$	$1/N_C^2$
T	$-\frac{g_A}{4m_N^3}\{v_T^+(k_2)\tau_{\pm}^1(\vec{\sigma}^1\vec{k}_2^2 - \vec{k}_2\vec{\sigma}^1\cdot\vec{k}_2)$ $+v_T^-(k_2)\tau_{\pm}^2[4\vec{\sigma}^2\vec{P}_1\cdot\vec{P}_2 - 4\vec{P}_2\vec{\sigma}^2\cdot\vec{P}_1 - \vec{\sigma}^2\vec{P}_1^2 + \frac{3}{4}\vec{\sigma}^2\vec{k}_2^2 - \vec{k}_2\vec{\sigma}^2\cdot\vec{k}_2]$ $-v_T^-(k_2)(\vec{\tau}^1\times\vec{\tau}^2)_{\pm}[4\vec{\sigma}^1\vec{P}_1\cdot\vec{\sigma}^2\times\vec{P}_2 - 4\vec{\sigma}^2\times\vec{P}_2\vec{\sigma}^1\cdot\vec{P}_1$ $+3\vec{\sigma}^1\times\vec{P}_1\vec{\sigma}^2\cdot\vec{P}_1 - \frac{1}{4}\vec{\sigma}^1\times\vec{k}_2\vec{\sigma}^2\cdot\vec{k}_2 + \vec{\sigma}^2\times\vec{P}_1\vec{\sigma}^2\cdot\vec{P}_1$ $-\frac{3}{4}\vec{\sigma}^2\times\vec{k}_2\vec{\sigma}^1\cdot\vec{k}_2 - \vec{P}_1\vec{\sigma}^1\times\vec{\sigma}^2\cdot\vec{P}_1 - \frac{1}{4}\vec{k}_2\vec{\sigma}^1\times\vec{\sigma}^2\cdot\vec{k}_2\} + (1 \leftrightarrow 2)$	$1/N_C^2$
A	$-\frac{g_A}{4m_N^3}\{v_A^-(k_2)\tau_{\pm}^2(3\vec{\sigma}^2\vec{P}_1^2 - \frac{1}{4}\vec{\sigma}^2\vec{k}_2^2 - 4\vec{P}_1\vec{\sigma}^2\cdot\vec{P}_1$ $-v_A^-(k_2)(\vec{\tau}^1\times\vec{\tau}^2)_{\pm}[4\vec{\sigma}^1\times\vec{P}_1\vec{\sigma}^2\cdot\vec{P}_2 - \vec{\sigma}^1\times\vec{P}_1\vec{\sigma}^2\cdot\vec{P}_1$ $+ \frac{3}{4}\vec{\sigma}^1\times\vec{k}_2\vec{\sigma}^2\cdot\vec{k}_2 - 3\vec{\sigma}^2\times\vec{P}_1\vec{\sigma}^1\cdot\vec{P}_1 + \frac{1}{4}\vec{\sigma}^2\times\vec{k}_2\vec{\sigma}^1\cdot\vec{k}_2$ $-\vec{P}_1\vec{\sigma}^1\times\vec{\sigma}^2\cdot\vec{P}_1 - \frac{1}{4}\vec{k}_2\vec{\sigma}^1\times\vec{\sigma}^2\cdot\vec{k}_2\} + (1 \leftrightarrow 2)$	$1/N_C^2$

Table 8: Axial exchange current operators with the highest N_C scaling factors associated with the interaction components.

The axial charge operator of a single nucleon

$$A_{\pm}^0 = -g_A \frac{\vec{\sigma} \cdot \vec{p}}{m_N} \tau_{\pm}, \quad (23)$$

is readily seen to be of order N_C^0 by means of the relations (9). The main axial exchange charge operators listed in Table 6 are in comparison of order N_C^{-1} . This may be seen by referring to the operator scaling relations (9).

There is no axial exchange charge operator that would be directly proportional to the pseudoscalar invariant potential component. There is however an axial exchange charge operator, which is of order N_C^0 , and which arises from the long range pion exchange interaction, which gives rise to the bulk of the pseudoscalar interaction component. This axial charge operator was derived in ref. [26], and has the form

$$A_{\pm}^0(\pi) = \frac{g_A}{4\pi} \left(\frac{m_{\pi}}{2f_{\pi}} \right)^2 (\vec{\sigma}^1 + \vec{\sigma}^2) \cdot \vec{r}_{12} (\vec{\tau}^1 \times \vec{\tau}^2)_{\pm} \left(1 + \frac{1}{m_{\pi} r_{12}} \right) \frac{e^{-m_{\pi} r_{12}}}{m_{\pi} r_{12}}. \quad (24)$$

Because the pion decay constant $f_{\pi} \sim N_C$ in the large N_C limit it is readily seen by the relations (9) that this operator is of order N_C^0 .

The contributions of the two-nucleon exchange current terms to the axial charge operator are conventionally expressed in the form of an effective single nucleon axial charge operator. The exchange current contributions then represent nuclear enhancement factors of the single nucleon axial charge operator (23). Experimentally this enhancement factor is a factor 2 as measured by first forbidden β transitions of nuclei in the lead region [27]. About one half of this enhancement may be attributed to the pion exchange operator (24), which is natural as its order in the $1/N_C$ expansion is the same as that of the single nucleon operator (23). The bulk of the remainder arises from the isospin independent scalar contribution in Table 9 [3]. While the order of this term is $1/N_C$, it gives rise to a direct term matrix element, which involves an additional factor of N_C in the nucleon density, and therefore its contribution to the “effective” single nucleon matrix element is also of order N_C^0 .

6 Discussion

This analysis of 4 commonly employed phenomenological nucleon-nucleon interaction models reveals that the structure of their components is completely consistent with their corresponding dependence on N_C (or $1/N_C$). For all the interaction models the isospin dependent pseudoscalar components, which contain the long range pion exchange interaction, have volume integrals that are larger than those of any other interaction component by more than an order of magnitude.

Fermi invariant	A^0	v_j^+	v_j^-
S	$\frac{g_A}{m_N^2} [v_S^+(k_2)\tau_\pm^1 + v_S^-(k_2)\tau_\pm^2] \vec{\sigma}^1 \cdot \vec{P}_1 + (1 \leftrightarrow 2)$	N_C^{-1}	N_C^{-3}
V	$\frac{g_A}{m_N^2} \{ [v_V^+(k_2)\tau_\pm^1 + v_V^-(k_2)\tau_\pm^2] [\vec{\sigma}^1 \cdot \vec{P}_2 + \frac{1}{2} \vec{\sigma}^1 \times \vec{\sigma}^2 \cdot \vec{k}_2]$ $+ \frac{i}{2} v_V^-(k_2) (\vec{\tau}^1 \times \vec{\tau}^2)_\pm \vec{\sigma}^1 \cdot \vec{k}_2 \} + (1 \leftrightarrow 2)$	N_C^{-1}	N_C^{-2}
T	$\frac{g_A}{m_N^2} \{ [v_T^+(k_2)\tau_\pm^1 + v_T^-(k_2)\tau_\pm^2] [\vec{\sigma}^2 \cdot \vec{P}_1 + \frac{i}{2} \vec{\sigma}^1 \times \vec{\sigma}^2 \cdot \vec{k}_2]$ $+ i v_T^-(k_2) (\vec{\tau}^1 \times \vec{\tau}^2)_\pm [\frac{1}{2} \vec{\sigma}^1 \cdot \vec{k}_2 + i \vec{\sigma}^1 \times \vec{\sigma}^2 \cdot \vec{P}_2] \} + (1 \leftrightarrow 2)$	N_C^{-2}	N_C^{-1}
A	$\frac{g_A}{m_N^2} \{ [v_A^+(k_2)\tau_\pm^1 + v_A^-(k_2)\tau_\pm^2] \vec{\sigma}^2 \cdot \vec{P}_2$ $+ i v_A^-(k_2) (\vec{\tau}^1 \times \vec{\tau}^2)_\pm [\frac{1}{2} \vec{\sigma}^2 \cdot \vec{k}_2 + i \vec{\sigma}^1 \times \vec{\sigma}^2 \cdot \vec{P}_1] \} + (1 \leftrightarrow 2)$	N_C^{-3}	N_C^{-1}

Table 9: Axial exchange charge operators and highest N_C scaling factors associated with the interaction components. The fractions of the imparted momentum q to the two nucleons are denoted k_1 and k_2 respectively.

This interaction component also scales with the largest power of N_C (N_C^3) (Table 2). As this interaction component shows very little variation between 3 of the 4 interaction models considered, it may be viewed as well determined, if not completely settled in form (Fig. 10).

The interaction components that follow the isospin dependent pseudoscalar interaction in strength are the isospin independent scalar and vector interactions, which scale as N_C . These interaction components are those responsible for nuclear binding (v_S^+) and short range repulsion between nucleons (v_V^-). For distances shorter than 0.4 fm the variation in form of these interaction components between the 4 considered phenomenological interaction models is substantial.

Next in strength are the isospin dependent scalar and vector interactions, which also scale as N_C . The volume integrals of these interaction components are about half as large as those of the corresponding isospin independent interactions. A new finding is that these two interaction components have to have equal magnitude and opposite sign in the large N_C limit.

The tensor and axial vector interaction components are very weak for all the considered phenomenological interaction models. For the isospin independent tensor and axial vector components this is completely consistent with their N_C dependence, which is $1/N_C$. The smallness of the isospin dependent tensor and axial vector exchange interaction components cannot be explained by their N_C dependence alone, as they scale as N_C . In meson exchange models the smallness of v_T^- is however natural, as it arises from tensor coupled vector mesons, and contains an overall factor $1/m_N^2$ (13).

The least well understood interaction component is the isospin independent pseudoscalar interaction (Fig. 9). This component, which scales as N_C^1 , is vanishingly small in two of the considered phenomenological interactions and even stronger than v_V^+ in the other two. The longest range part of this interaction component arises from η meson exchange. The large variation between the phenomenological interaction models for this interaction component reflects the continuing uncertainty concerning the strength of the η –nucleon coupling. The *V18* and *Paris* interaction models have strong “effective” η nucleon couplings, which correspond to the old pseudoscalar $SU(3)$ coupling model by which $g_{\eta NN} = g_{\pi NN}/\sqrt{3}$.

The present results for the N_C dependence of the exchange currents follow those of the corresponding interactions. In the case of the electromagnetic interaction current the result that they are smaller by $1/N_C^2$ than the corresponding interactions fits with the phenomenological finding that electromagnetic exchange current contributions to nuclear observables typically represent effects of the order 10%, the exceptions being situations where single nucleon current matrix elements are suppressed [24].

The interaction current contributions to the axial charge operator are known to be significant in comparison with the axial charge operators of the single nucleons [27]. This feature also emerges from the $1/N_C$ expansion, as the axial exchange charge operators scale in $1/N_C$ as the single nucleon charge operator, or with one more power of $1/N_C$.

The overall conclusion is thus that the ordering of nuclear interactions and interaction current

operators by their dependence on $1/N_C$ corresponds well with their significance ordering based on empirical evidence and phenomenological analysis.

Acknowledgments

The hospitality of the W. K. Kellogg Radiation Laboratory of the California Institute of Technology is gratefully acknowledged. I am indebted to C. Helminen for a careful reading of the manuscript. Research supported in part by the Academy of Finland by grant 54038.

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Figure captions

Fig. 1 Isospin independent scalar potential coefficients for the *V18* [9], the *CD – Bonn* [10], the *Nijmegen(93)* [11] and the *Paris* [12] interaction models.

Fig. 2 Isospin dependent scalar potential coefficients for the *V18* [9], the *CD – Bonn* [10], the *Nijmegen(93)* [11] and the *Paris* [12] interaction models.

Fig. 3 Isospin independent vector potential coefficients for the *V18* [9], the *CD – Bonn* [10], the *Nijmegen(93)* [11] and the *Paris* [12] interaction models.

Fig. 4 Isospin dependent vector potential coefficients for the *V18* [9], the *CD – Bonn* [10], the *Nijmegen(93)* [11] and the *Paris* [12] interaction models.

Fig. 5 Isospin independent tensor potential coefficients for the *V18* [9], the *Nijmegen(93)* [11] and the *Paris* [12] interaction models.

Fig. 6 Isospin dependent tensor potential coefficients for the *V18* [9], the *CD – Bonn* [10], the *Nijmegen(93)* [11] and the *Paris* [12] interaction models.

Fig. 7 Isospin independent axial vector potential coefficients for the *V18* [9] and the *Paris* [12] interaction models.

Fig. 8 Isospin dependent axial vector potential coefficients for the *V18* [9], the *Nijmegen(93)* and the *Paris* [12] interaction models.

Fig. 9 Isospin dependent pseudoscalar potential coefficients for the *V18* [9], the *Nijmegen(93)* [11] and the *Paris* [12] interaction models.

Fig. 10 Isospin dependent pseudoscalar potential coefficients for the *V18* [9], the *CD – Bonn* [10], the *Nijmegen(93)* [11] and the *Paris* [12] interaction models.

Fig. 11 (a) Point coupling of an external field to one nucleon and a nucleon-nucleon interaction, (b) Coupling to the charge exchanged by a nucleon-nucleon interaction.